M.Stat I Year

Statistical Inference

End-Term Examination

Answer any 6 questions

Total Marks : 60 Duration : 3 hours

(1) Let $\Theta = \{0, 1\}, A = \{0, 1\}$ and let the loss function be $L(0, 0) = L(1, 1) = 0$, $L(1, 0) = 1, L(0, 1) = 1$. Suppose the statistician observes the random variable X with the discrete distribution $P(X = x| \theta) = 2^{-k}$, if $x = k - \theta$ for $k = 1, 2, ...$

(a) Describe the set of all non randomised decision rules of the statistician. Plot the risk set S in the plane.

(b) Find a minimax decision rule..

(c) Find a least favourable distribution.

(2) Let $\Theta = (0, 1)$ and let $A = [0, 1]$. Let $L(\theta, a) = (\theta - a)^2$. Let the distribution of X be binomial with n trials and probabilty θ of success, $f_X(x|\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}, x = 0, 1, 2...n$.

Let the prior distribution of θ be the beta distribution $B(\alpha, \beta)$.

(a) Find the posterior distribution of θ given $X = x$.

(b) Determine the Bayes rule.

(c) Show that the usual MLE is not a Bayes rule.

(d) Show that d is a limit of Bayes rule.

(e) Show that d is extended Bayes .

(3) (a) Show that if a minimal complete class exists, it consists of exactly the admissible rules.

(b) If for a given prior distribition τ a Bayes rule with respect to τ is unique upto equivalence, this bayes rule is admissible.

(c) If δ is admissible and Θ is finite, then δ is bayes (with respect to some prior distribution)

(4) Let Θ consists of two points $\{\theta_1, \theta_2\}$ and $A = [0, 1]$ and let the loss function be $L(\theta_1, a) = a^2$. $L(\theta_2, a) = 1 - a$. Note that this loss is convex in 'a' for each $\theta \in \Theta$. A coin is tossed once . The probability of heads is 1/3 if θ_1 is the true state of nature and 2/3 if θ_2 is the true state of nature .

(a) Represent the class D of decision rules as a subset of the plane.

(b) Find $R(\theta_1, (x, y))$ and $R(\theta_2, (x, y))$ for $x, y \in D$.

(c) Find the class of all non-randomised bayes rules for a given prior $(p, 1-p)$ which selects $\theta = 1/3$ with probability p. Show that the bayes risk is $r(p, d) = p(2x^2 + y^2)/3 + (1 - p)[(1 - x) + 2(1 - y)]/3$ for $(x, y) \in D$.

(d) Find a minimax rule among the class of bayes rules.

(5) Let $f(x|\theta) = exp(x - \theta)/(1 + exp(x - \theta))^2 \ -\infty < x < \infty, -\infty < \theta < \infty$.

(a) Show that this family has an MLR.

(b) Based on one observation X, find the most pwerful size α test of $H_0 : \theta = 0$ versus $H_1 : \theta = 1$. For $\alpha = .2$. find the size of the type II error.

(c) Show that the test in part b) is UMP of its size α test for testing $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$.

(6) (a) Let $X_1, X_2, ...X_n$ be a random sample from the laplace pdf $f_\theta(x) = 1/2exp-(|x-\theta|)$. For $n \geq 2$, Show that a UMP size α test of $H_0 : \theta \leq 0$ against $H_1 : \theta > 0$ does not exist. Find the form of the locally most powerful test.

(b) Let X be a geometric random variable with mean 1/p. Find a UMP unbiased test of size α for $H_0 : p = p_0$ against $H_1 : p \neq 0$, where $p_0 \in (0, 1)$ is given.

(7) Let $X_1, X_2, ...X_n$ be iid Bernoulli(p).

(a) Find the most powerful test of size $\alpha = .0547$ of the hypothesis $H_0 : p = 1/2$ versus $H_1 : p = 1/4$. Find the power of the test .

(b) Suppose X is an observation from $beta(\theta, 1)$ pdf. For testing $H_0 : \theta \leq 1$ versus $H_1 : \theta > 1$. find the size and sketch the power function of the test that reject H_0 if $X > 1/2$. Is there a size α UMP test? If so find it.

(8) State and prove Karlin-Rubin's theorem.